Perceptual Measurements, Distances and Metrics

Jonathan Vacher

Joint work with : Pascal Mamassian

Imaging in Paris Seminar - Oct. 3rd 2023

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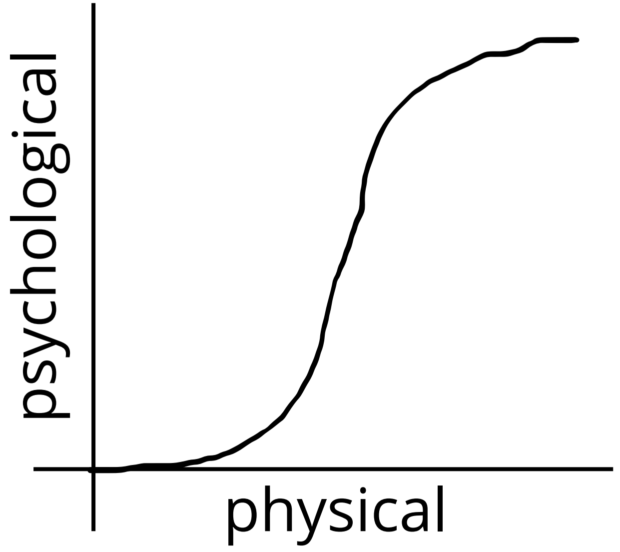

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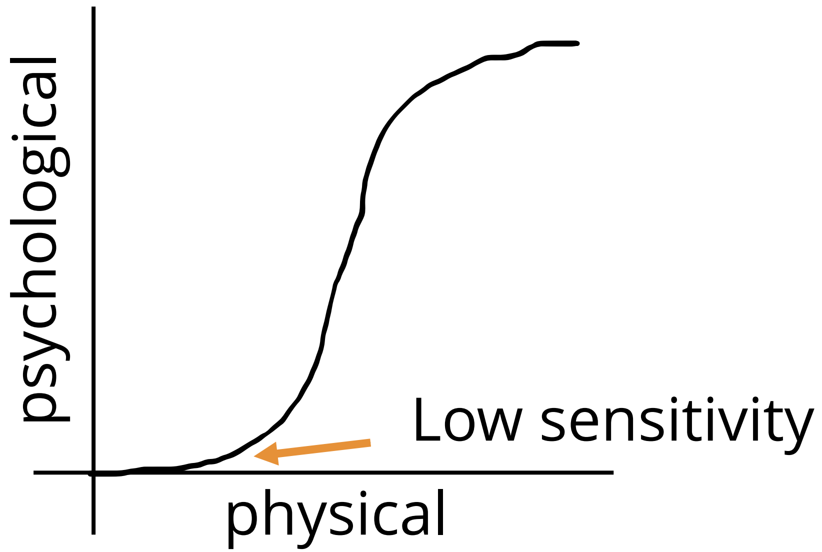

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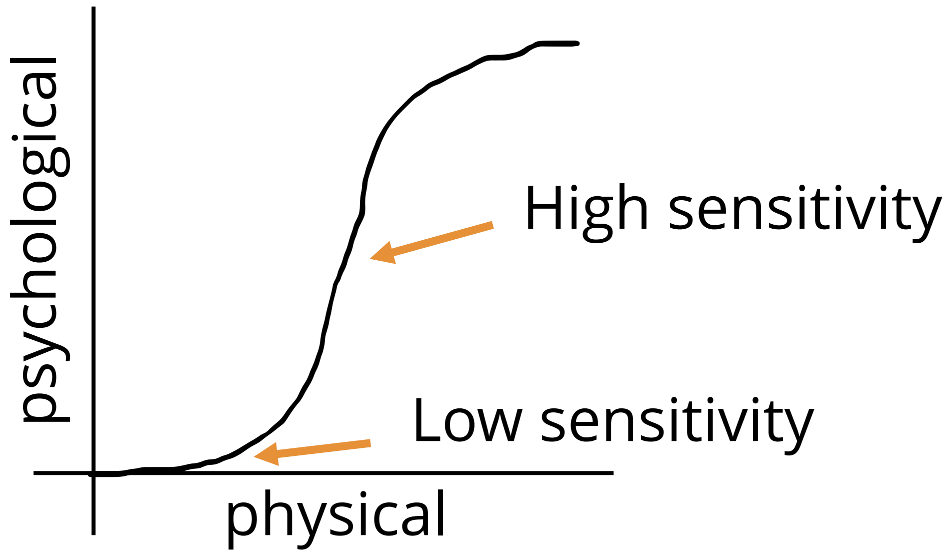

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- In psychophysics, less attention is dedicated to the encoding model
  - What are the measurements M ? Univariate ?
  - Is ψ the perceptual scale ? Prediction ?

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- Structural Similarity score has been proposed : based on local image statistics

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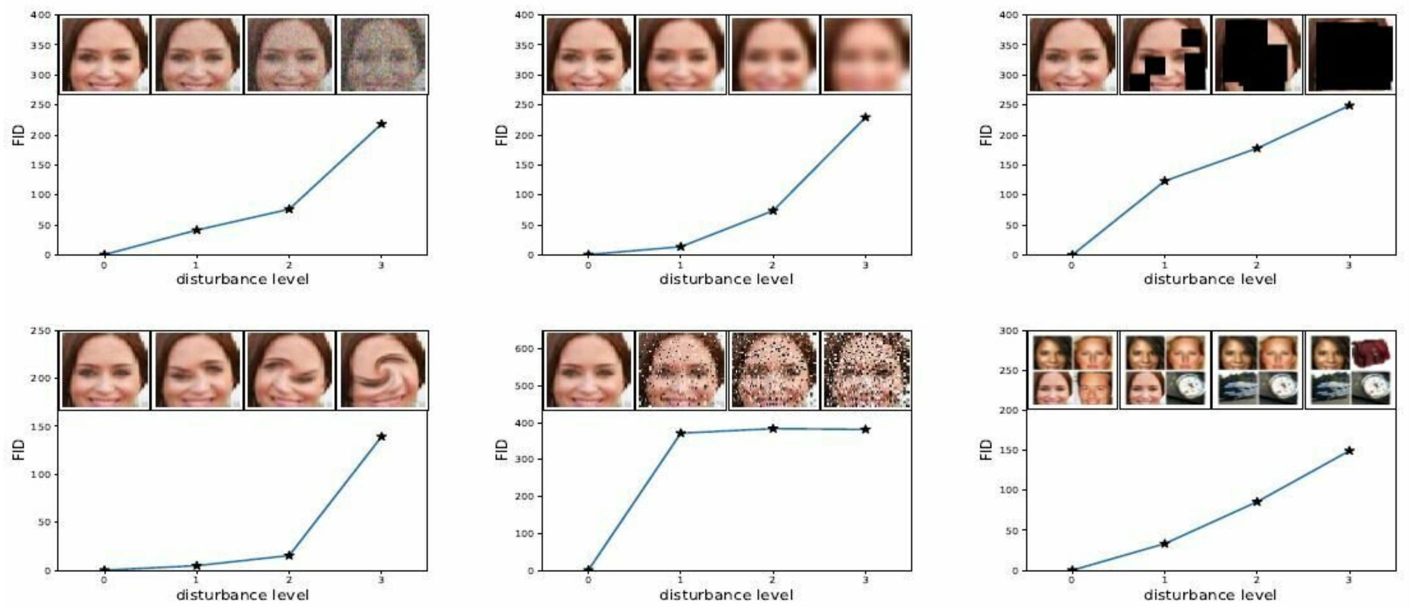

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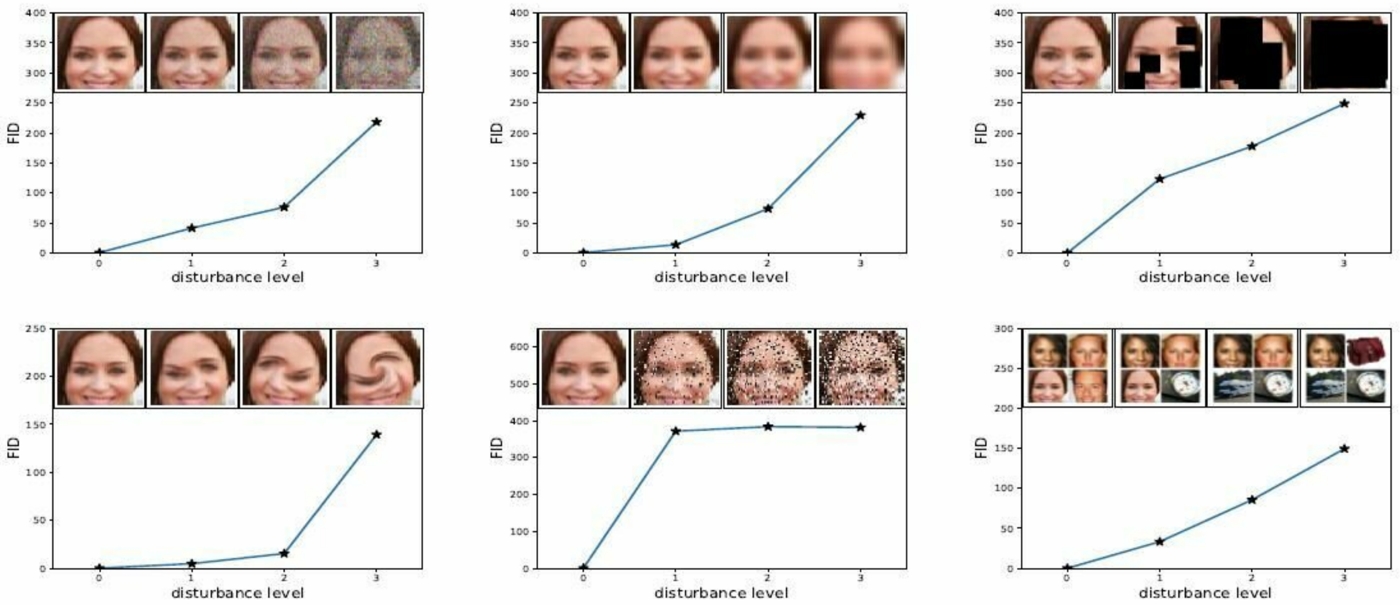

$$d((\mu_1, \Sigma_1), (\mu_2, \Sigma_2)) = ||\mu_1 - \mu_2||^2 + \mathrm{Tr}(\Sigma_1 + \Sigma_2 - 2(\Sigma_2^{\frac{1}{2}} \Sigma_1 \Sigma_2^{\frac{1}{2}})^{\frac{1}{2}})$$

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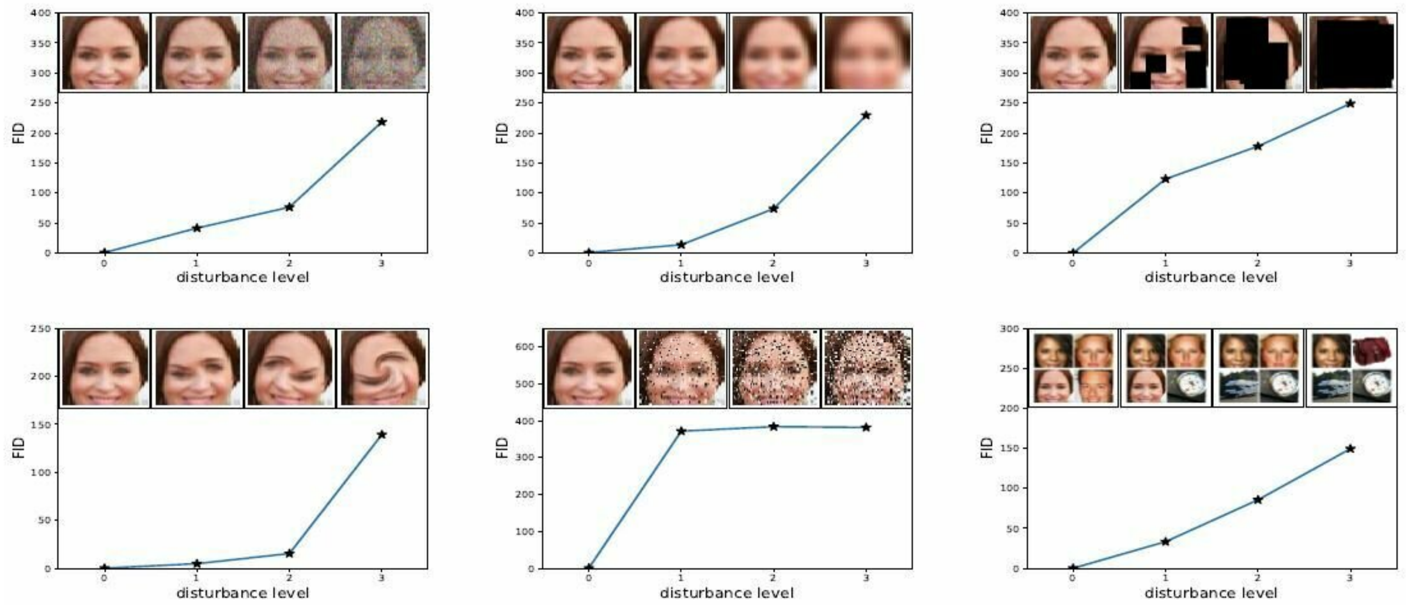

- Fréchet Inception Distance (FID) is used to compare generated to true datasets

$$\mathrm{d}((\mu_1, \Sigma_1), (\mu_2, \Sigma_2)) = ||\mu_1 - \mu_2||^2 + \mathbf{Tr}(\Sigma_1 + \Sigma_2 - 2(\Sigma_2^{\frac{1}{2}} \Sigma_1 \Sigma_2^{\frac{1}{2}})^{\frac{1}{2}})$$

- How about the geometry underlying those distances ?

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- Perceptual scales have not been embedded in recent theory of perception
- Theory are often vague and based on oversimple hypotheses (univariate framework)
- Perceptual distances are limited : moving towards perceptual metric

Stochastic Visual Stimulation

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We use two types of textures :

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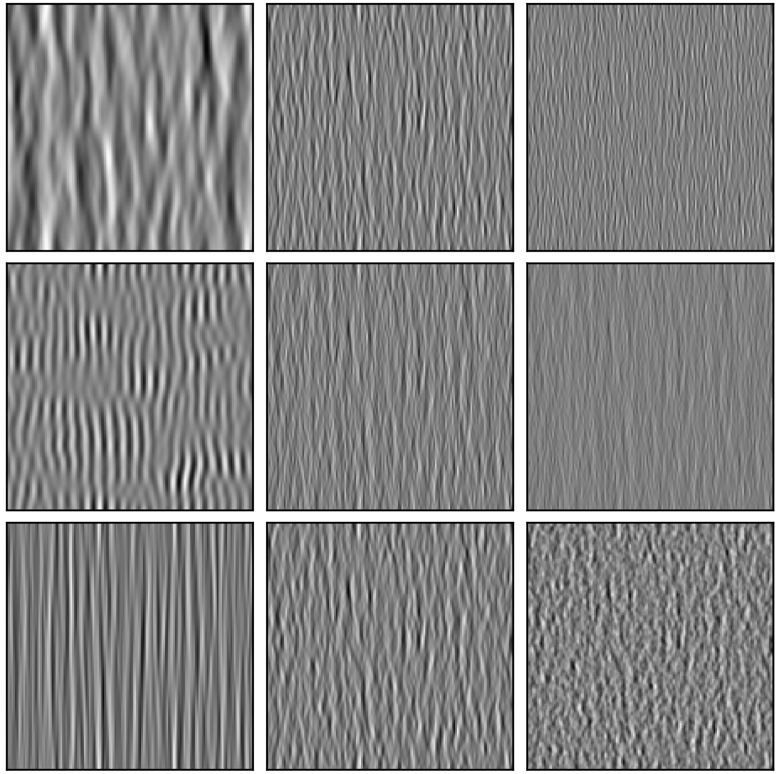
- Gaussian Random Field textures



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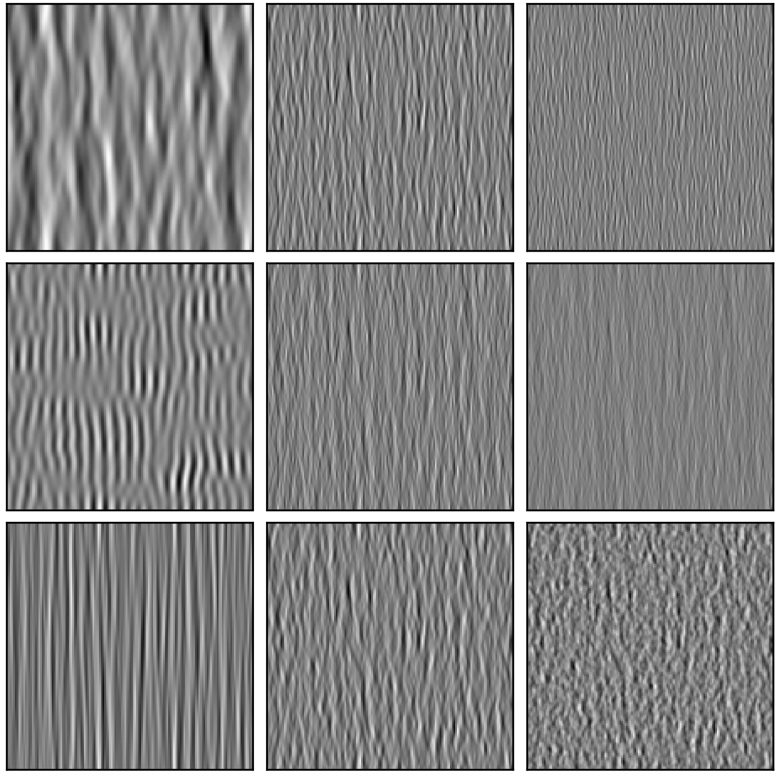
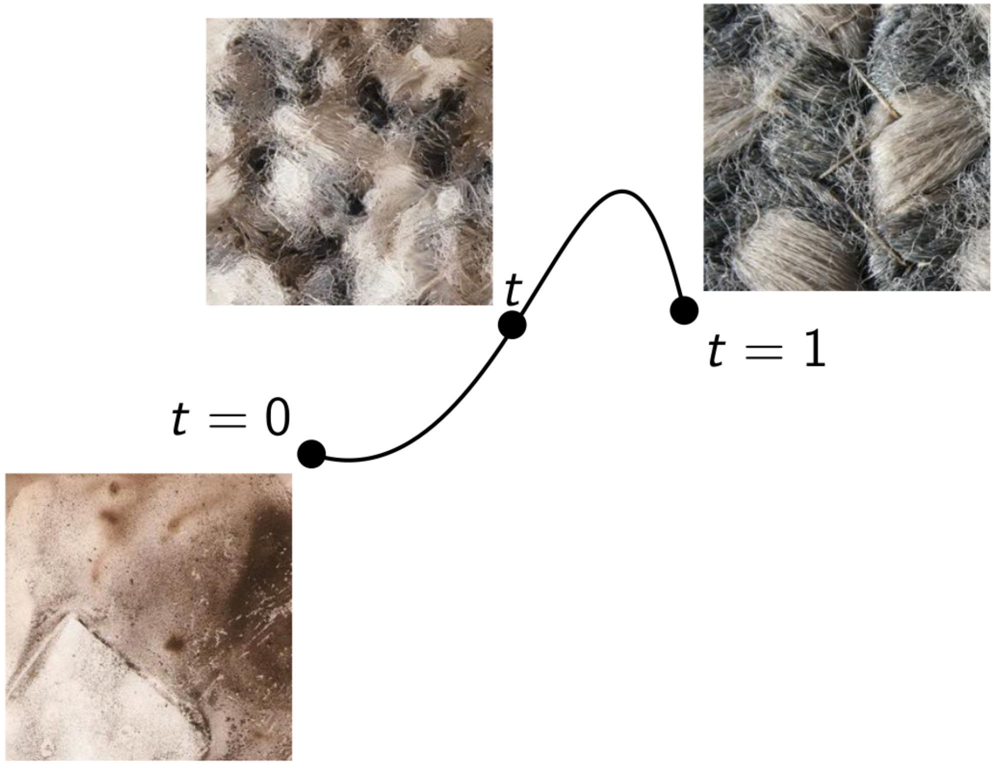


- Interpolated naturalistic textures



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► **Gabor function**

$$g_\sigma(x) = \frac{1}{2\pi}\cos(x \cdot \xi_0)e^{-\frac{\sigma^2}{2}||x||^2}$$

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$$\phi_{z,\theta}(x) = zR_{-\theta}(x)$$

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- **Shot Noise** $F_{\lambda,\sigma}(x) = \frac{1}{\sqrt{\lambda}}\sum_{k\in\mathbb{N}} g_\sigma(\phi_{Z_k,\Theta_k}(x - X_k))$ with $Z_k \sim \mathbb{P}_Z$ and $\Theta_k \sim \mathbb{P}_\Theta$

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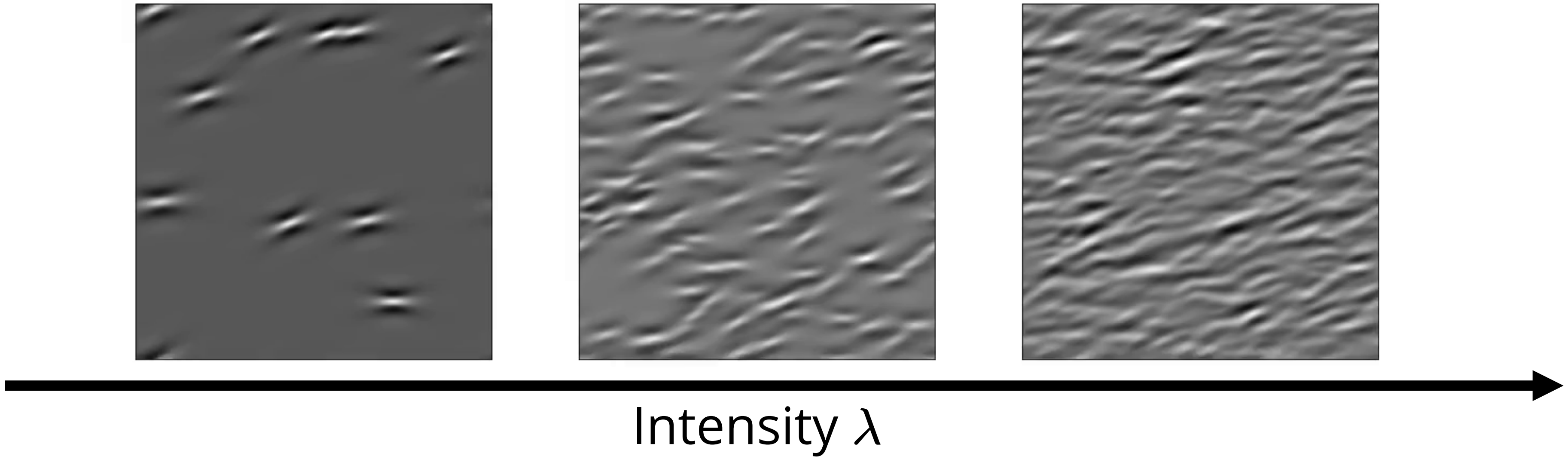

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Proposition 1 (*Convergence and Power Spectrum*). In the limit of infinite intensity ($\lambda \longrightarrow \infty$) and pure wave ($\sigma \longrightarrow 0$), $F_{\lambda,\sigma}$ converges towards a Gaussian field F with the following power spectrum for all $\xi \in \mathbb{R}^2$,

$$\hat{\gamma}(\xi) = \frac{1}{||\xi||^2}\mathbb{P}_Z(||\xi||)\mathbb{P}_\Theta(\angle\xi)$$

where $\xi = (||\xi|| \cos(\angle\xi), ||\xi|| \sin(\angle\xi))$.

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Naturalistic textures

Image u: natural texture sample. New example synthesis: minimize the following loss function

$$L(u,v) = \sum_{l=1}^{L} ||M_{N_l}(A_l^v) - \mu_l^u||^2 + ||C_{N_l}(A_l^v)^{\frac{1}{2}} - \Sigma_l^{u\,\frac{1}{2}}||_f$$

where $M_{N_l}(X)$ and $C_{N_l}(X)$: empirical mean and covariance of X (matrix of N_l vector samples) and A_l^v : activation of a CNN at layer l of texture v.

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Interpolated naturalistic textures

Given two textures u_1 and u_2, we synthesize an interpolated textures by minimizing

$$L_{\text{interp}}(u_1, u_2, s, v) = \; sL(u_1, v) + (1-s)L(u_2, v)$$

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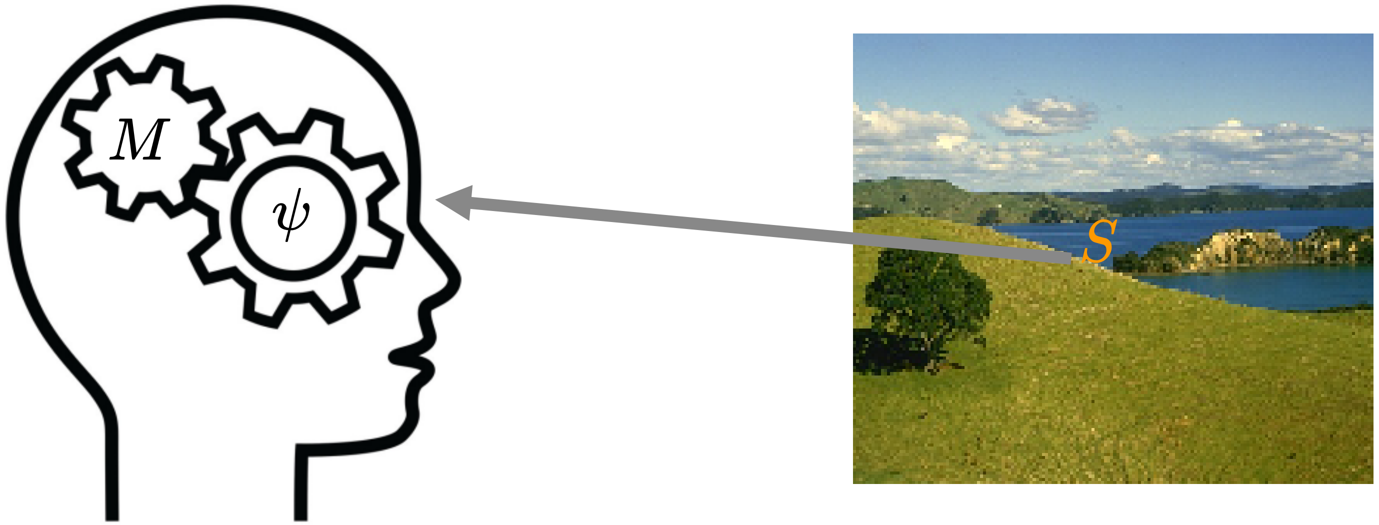
Thurstone Scale, Fisher Info. and MLDS



- An observer deduces the stimuli S from its measurement M

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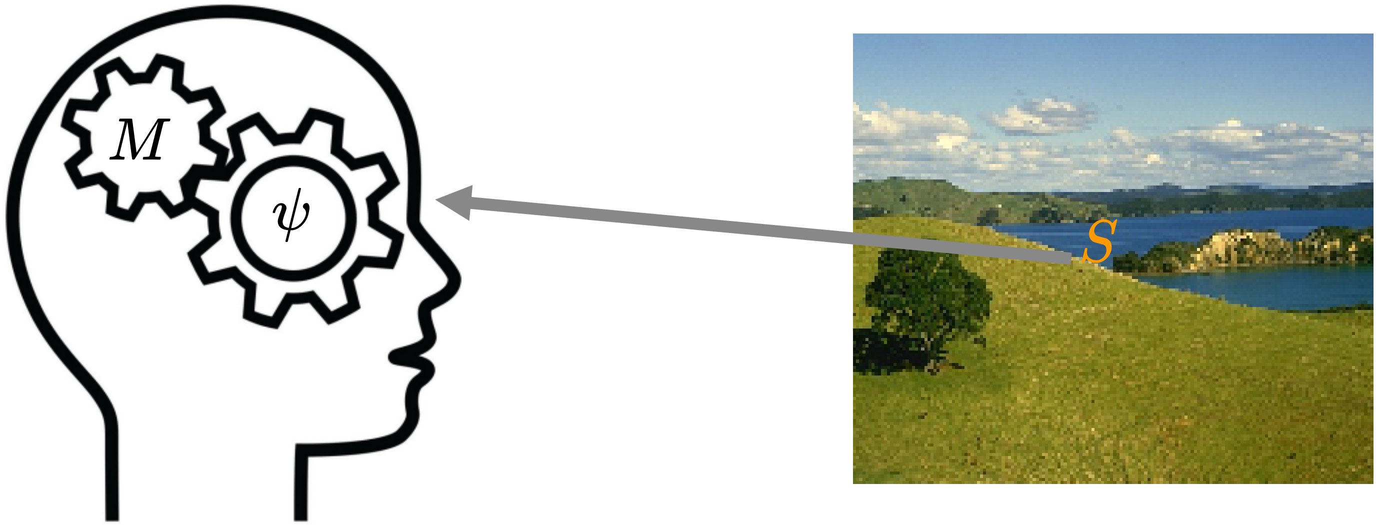
Encoding model of perception

$$M = R + N \text{ where } R = \psi(S) \text{ with } \psi : \mathbb{S} \to \mathbb{S}$$



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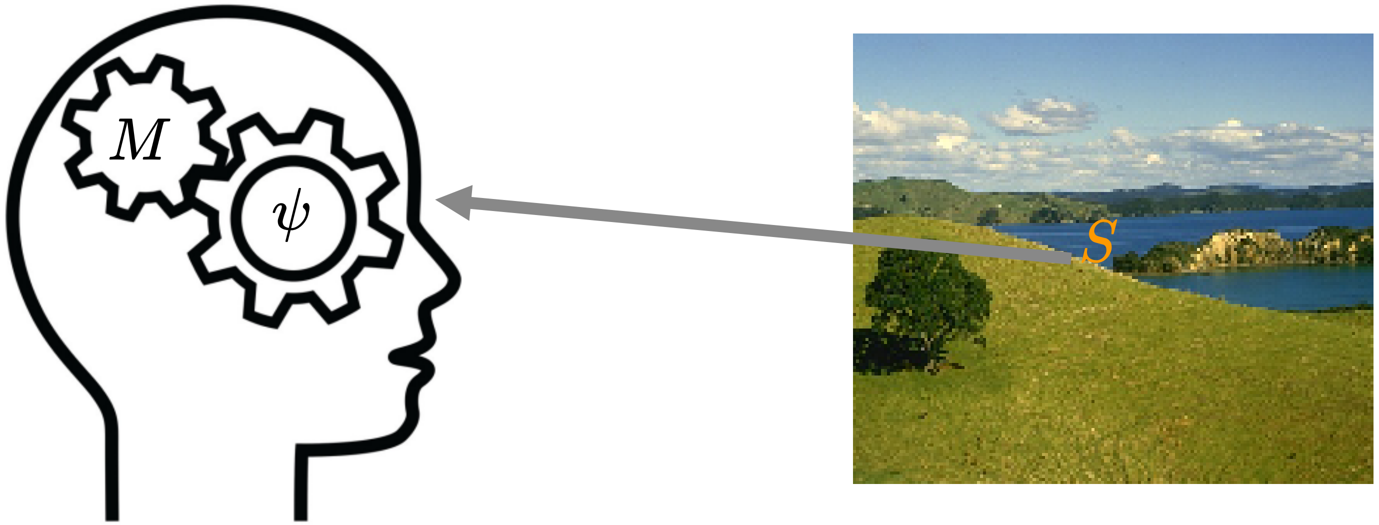

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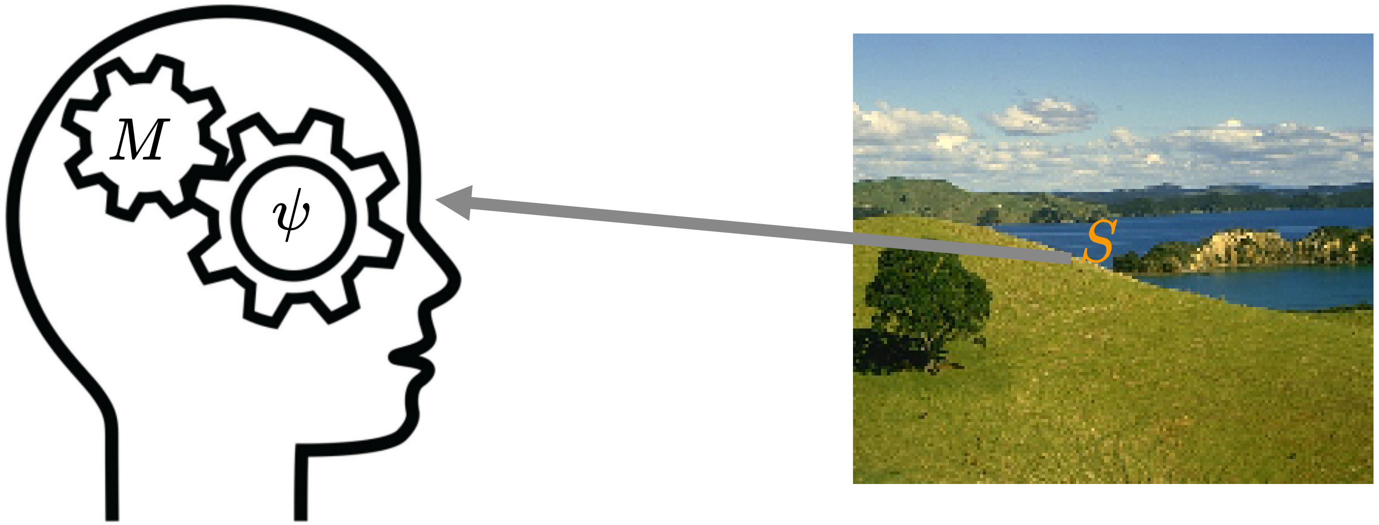

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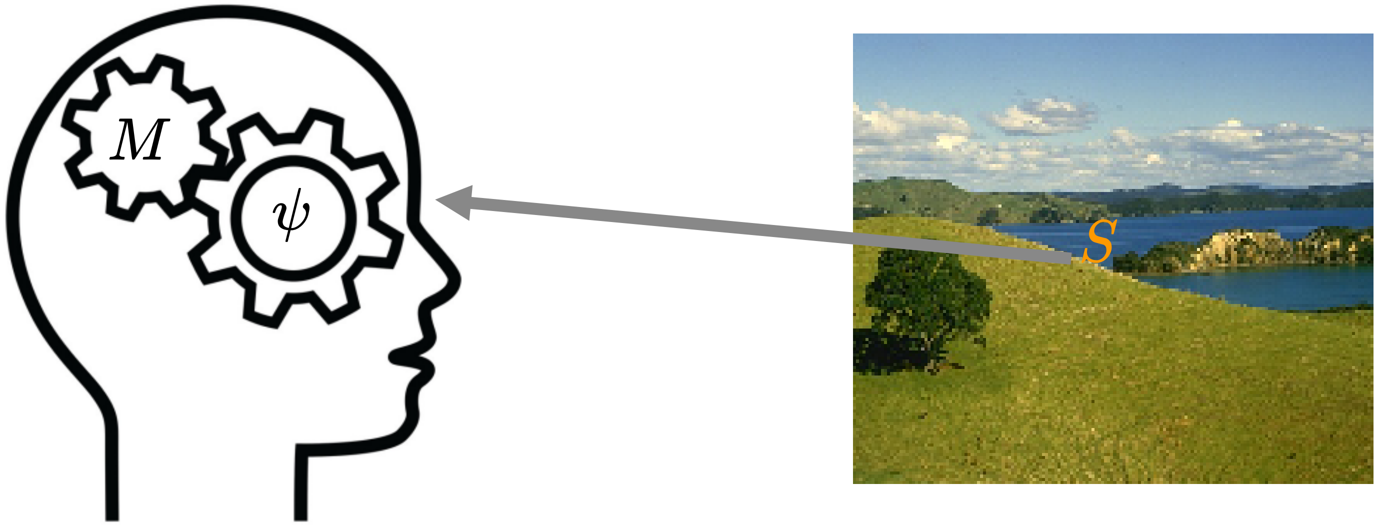
- N : Gaussian noise with constant variance σ^2 (Thurstone hypothesis)



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Definition 1 (*One-dimensional Fisher information*). Let X and Y be two random variables defined respectively on two abstract spaces X and Y and let $\mathbb{P}_{X|Y}$ be the conditional density of X knowing Y. The Fisher information carried by X about Y is a function $\mathcal{I} : Y \to \mathbb{R}$ defined for all $y \in Y$ by

$$\mathcal{I}_Y(y) = \mathbb{E}_{X|Y}\left(\left(\frac{\partial \log(\mathbb{P}_{X|Y})}{\partial y}(X, y)\right)^2\right).$$

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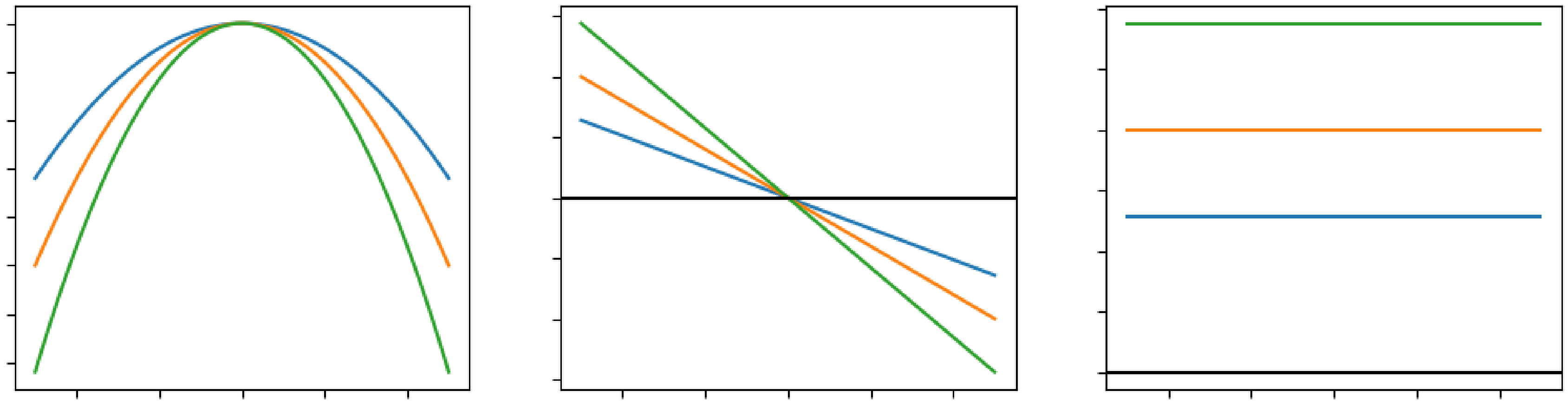

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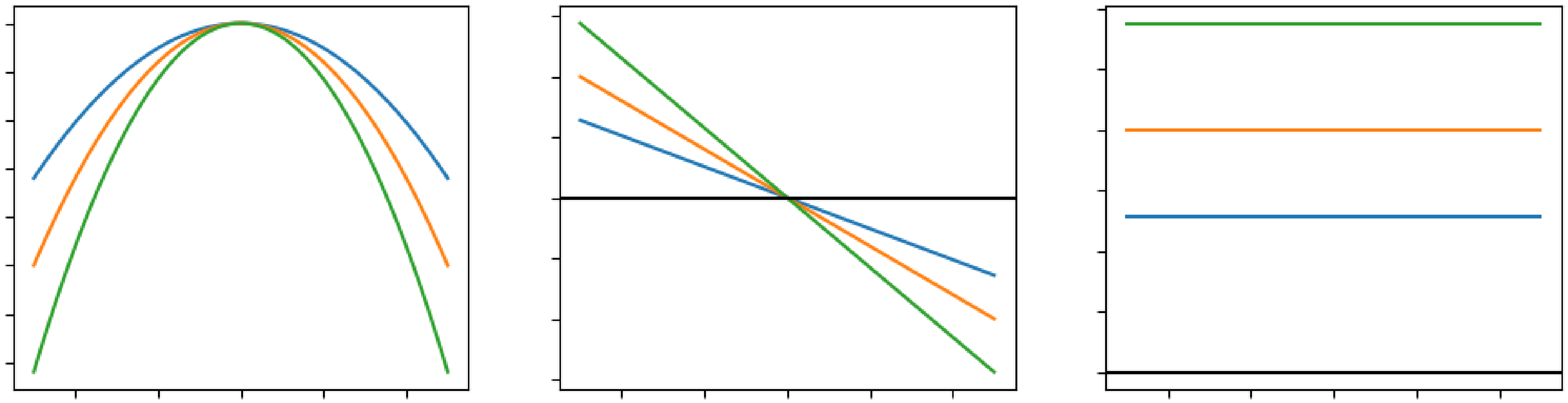
Log - Likelihood



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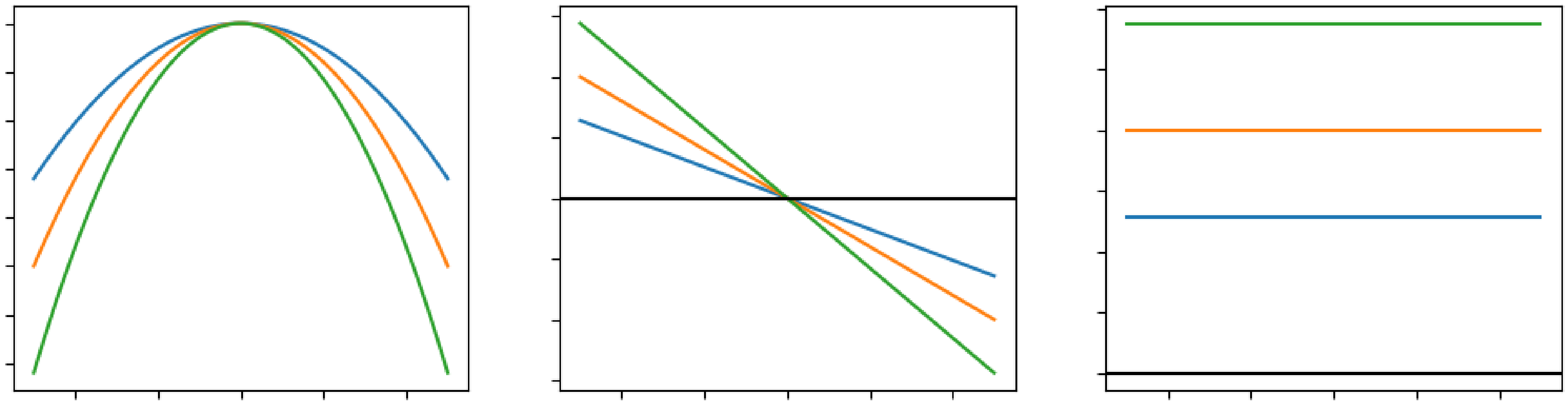
Log - Likelihood Score (gradient)



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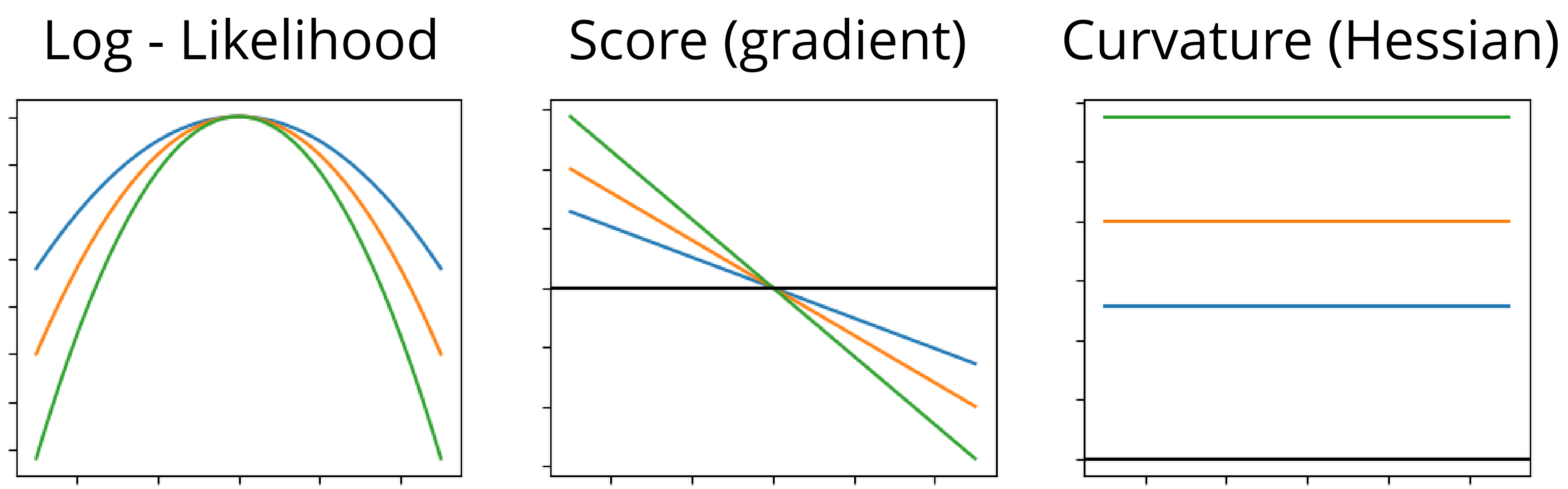

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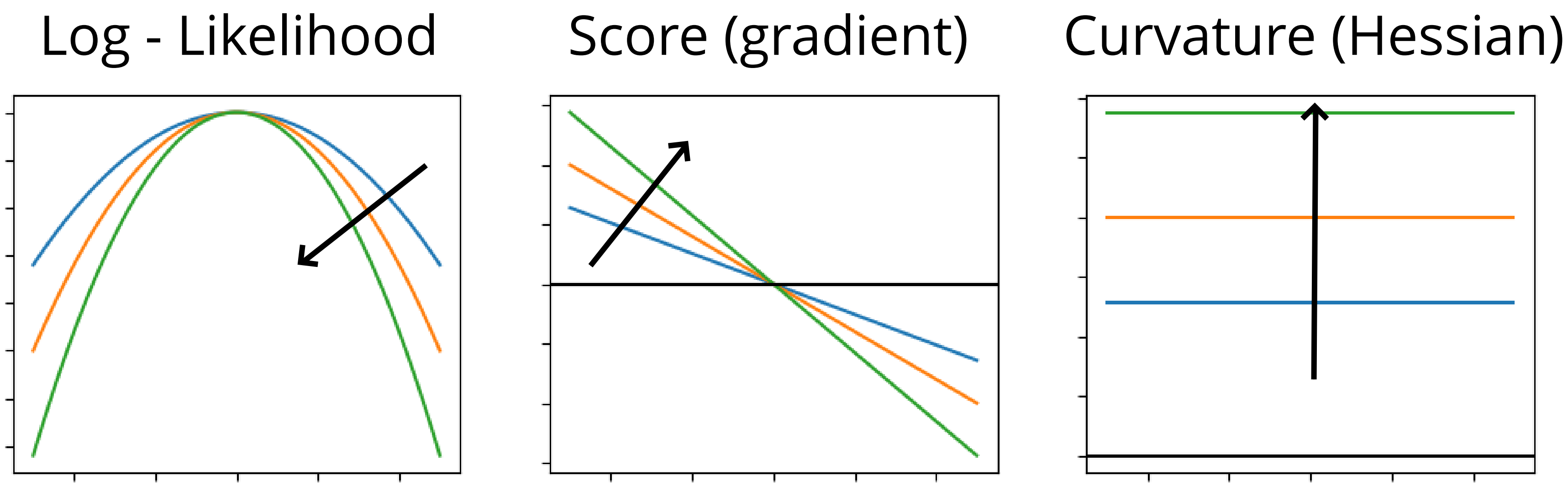

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Proposition 1. Assume $M = \psi(S) + N$, the internal Fisher information $\mathcal{I}_R$ is constant if and only if for all $s \in \mathbb{S}$ the psychological function ψ verifies

$$\psi(s) \propto \int_{s_{\text{init}}}^{s} \sqrt{\mathcal{I}_S(t)} \mathrm{d}t.$$

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Does it make any sense ?

Thurstone Scale, Fisher Info. and MLDS

- $\mathcal{I}_S$ and $\mathcal{I}_R$ are different, in fact they are related $\mathcal{I}_S(s) = \psi'(s)^2 \mathcal{I}_R(\psi(s))$.

Proposition 1. Assume $M = \psi(S) + N$, the internal Fisher information $\mathcal{I}_R$ is constant if and only if for all $s \in \mathrm{S}$ the psychological function ψ verifies

$$\psi(s) \propto \int_{s_{\mathrm{init}}}^{s} \sqrt{\mathcal{I}_S(t)}\mathrm{d}t.$$

Reformulation of Thurstone hypothesis.

- Internal Fisher information ($\mathcal{I}_R$) is constant.

Does it make any sense ?

- Yes ! An observer only accesses to its internal measurements. It makes senses to equalize the precision over all internal states R.

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- Samples along the stimulus domain $\{s_1, \ldots, s_d\} \in \mathbb{S}$

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- Suppose the existence of a perceptual scale ψ

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Observer model in the MLDS framework.
Comparison of relative differences

$$\Delta_{i,j,k} = |\psi(s_i) - \psi(s_j)| - |\psi(s_j) - \psi(s_k)| + N_{\text{mlds}}$$

where $N_{\text{mlds}} \sim \mathcal{N}(0, \sigma^2_{\text{mlds}})$.

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Experimental data : we get estimates $p_{i,j,k} \simeq \mathbb{E}(\Delta_{i,j,k} > 0)$

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In this case: MLDS $\Longleftrightarrow$ Encoding model and

$$\sigma^2_{\text{mlds}} = 4\sigma^2$$

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Fisher Info. of Image *vs* Local Features

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- Standard hypothesis in psychophysics : univariate representation

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$$\mathbb{P}_Z(z; z_0, b_Z) = \frac{2}{z b_Z \sqrt{\ln(2)\pi}} \exp\left(-\frac{4\left(\ln\left(\frac{z}{z_0}\right) - \frac{\ln(2)}{8} b_Z^2\right)^2}{\ln(2) b_Z^2} \right)$$

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The Fisher information carried by Z about Z_0 is $\mathcal{I}(z_0) = \frac{8}{z_0^2 b_Z^2 \ln(2)}$

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Then, the perceptual scale is $\psi(z_0) \propto \ln\left(\frac{z_0}{z_0^{(0)}}\right)$ (Weber-Fechner law)

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Fisher Info. of Image *vs* Local Features

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- Alternative hypothesis : use of the image itself (pixels)

The texture can be expressed for all $x \in \mathbb{R}^2$ and $s \in \mathbb{S}$, as

$$F(x, s) = \mu + \int_{\mathbb{R}^2} k(x - y, s)\mathrm{d}W(y)$$

where $k(\cdot, s) = \mathcal{F}^{-1}(\sqrt{\hat{\gamma}(\cdot, s)})$ and W is a classical Wiener process.

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The Fisher information carried by F about S is

$$\mathcal{I}_S(s) = \frac{1}{2}\int_{\mathbb{R}^2} \frac{1}{|\hat{\gamma}(\xi,s)|^2}\left|\frac{\partial\hat{\gamma}(\xi,s)}{\partial s}\right|^2 \mathrm{d}\xi = \frac{1}{2}\int_{\mathbb{R}^2} \left|\frac{\partial \ln(\hat{\gamma}(\xi,s))}{\partial s}\right|^2 \mathrm{d}\xi.$$

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Together with Proposition 1, this leads to the same prediction as the standard hypothesis.

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- The case of interpolated naturalistic textures

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If those activations are Gaussian then

$$A_k(s) = \mu_k(s) + \Sigma_k(s)N$$

where $N \sim \mathcal{N}(0, I_{d_k})$.

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The Fisher information carried by A about S is

$$\mathcal{I}_S(s) = \mu_k'(s)\Sigma_k(s)^{-1}\mu_k'(s) + \frac{1}{2}\mathrm{Tr}\left(\Sigma_k(s)^{-1}\Sigma_k'(s)\Sigma_k(s)^{-1}\Sigma_k'(s)\right)$$

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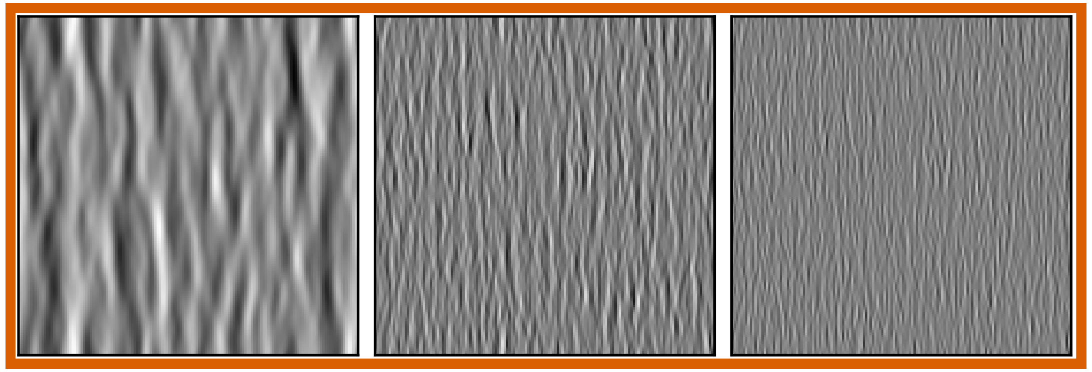
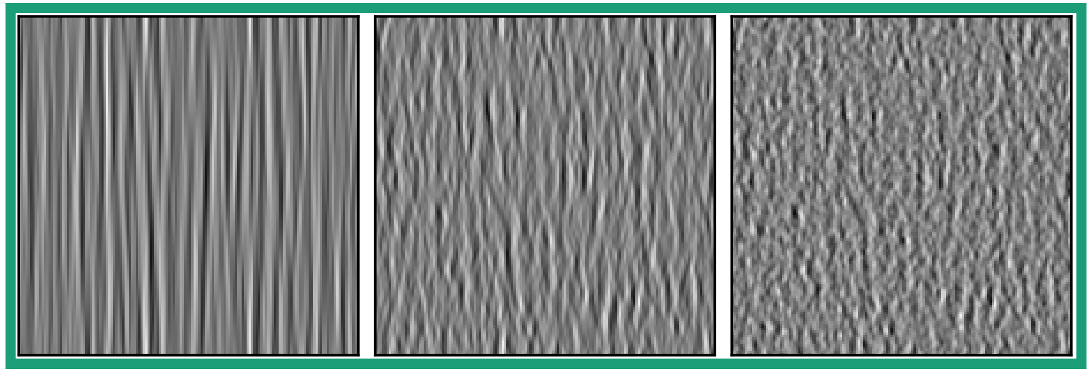
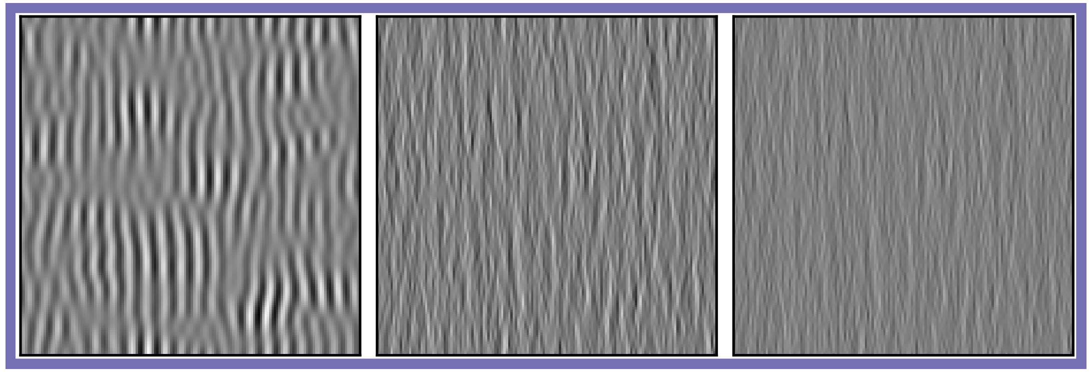
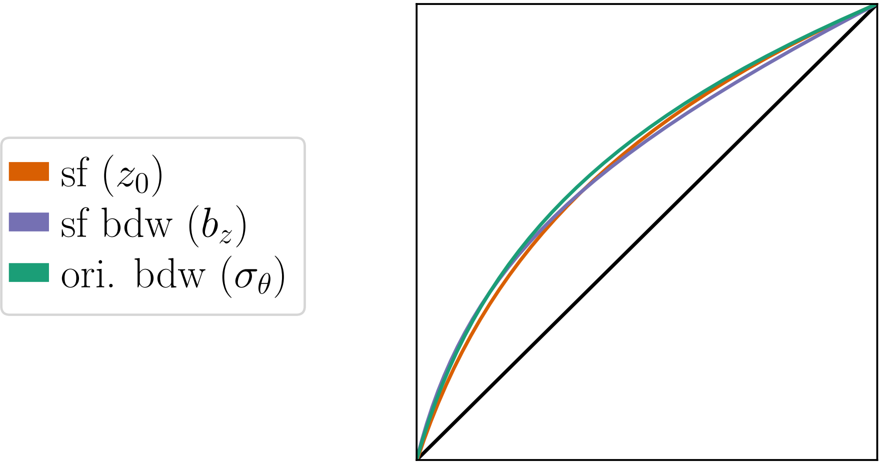
Predictions (Gaussian Textures)









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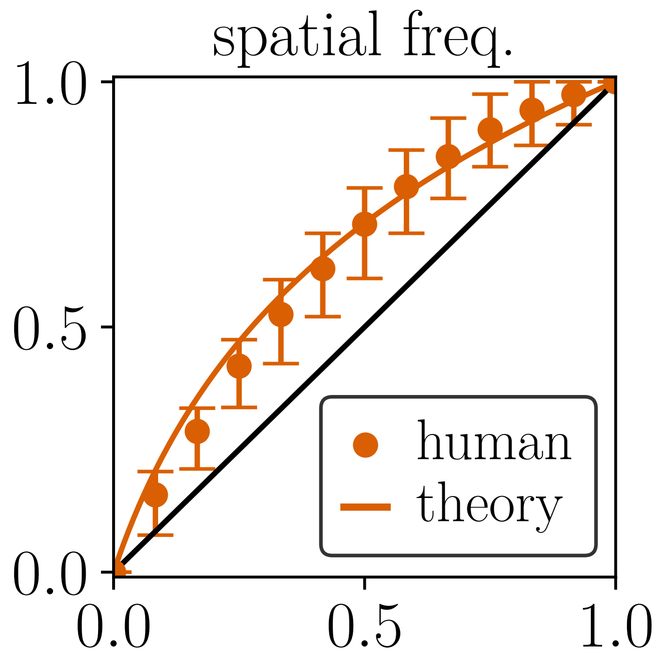

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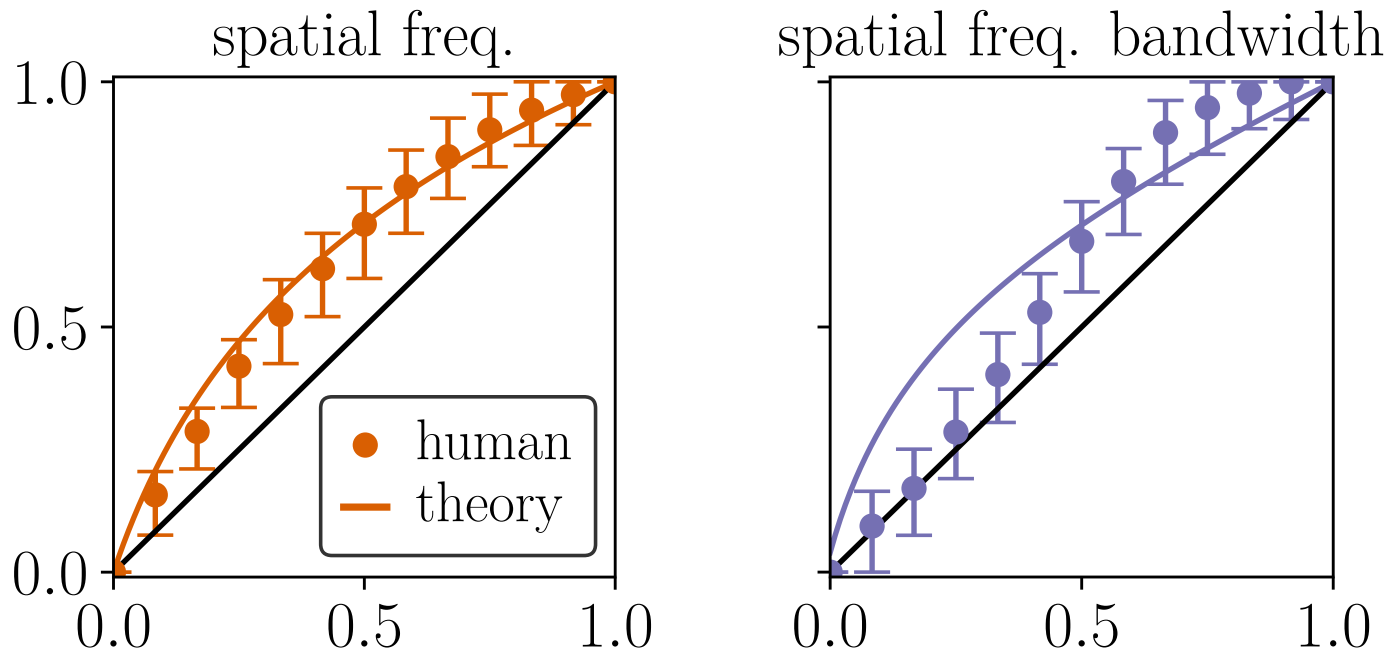

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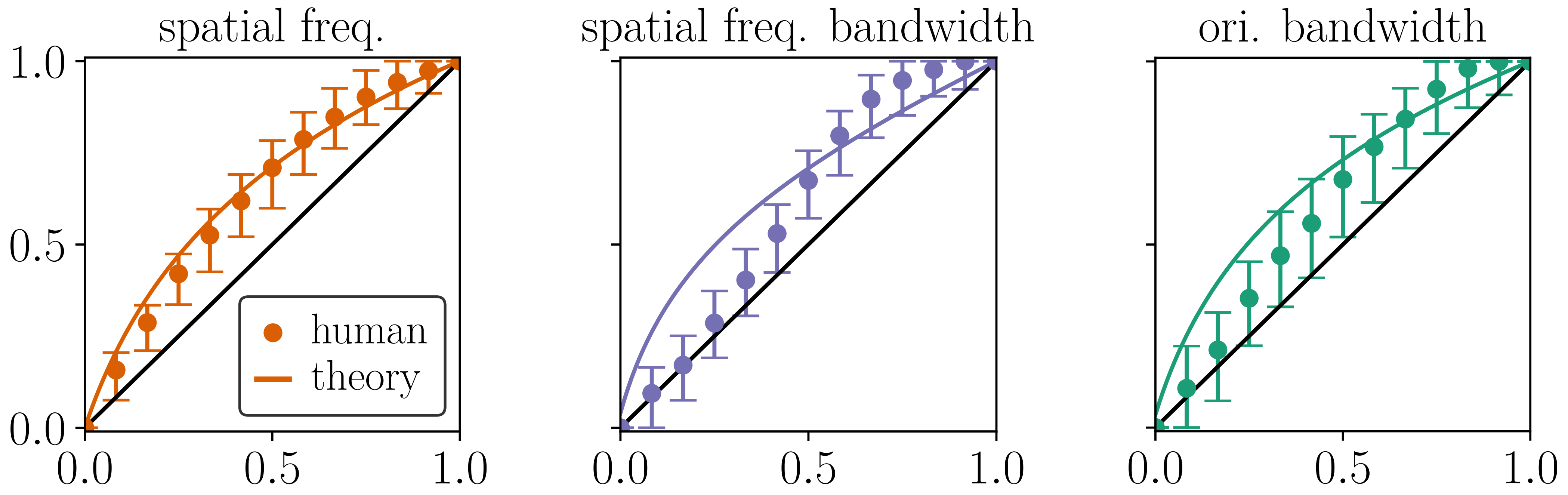

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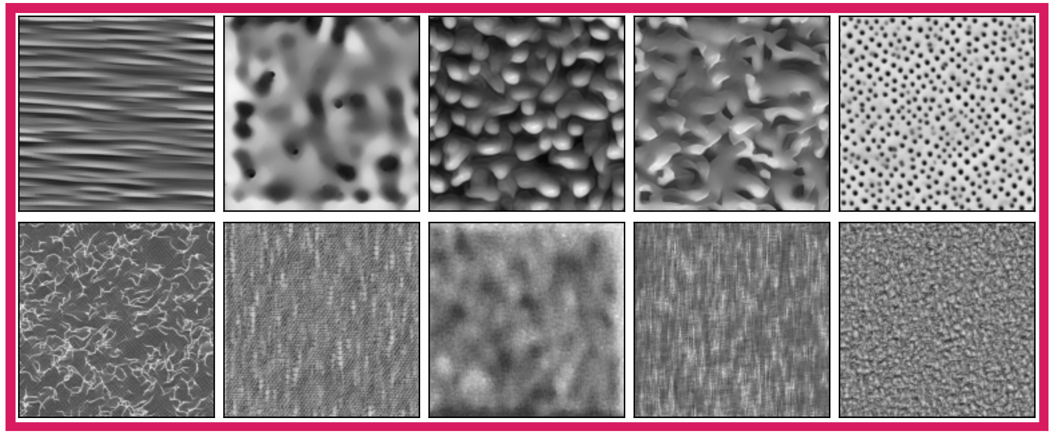
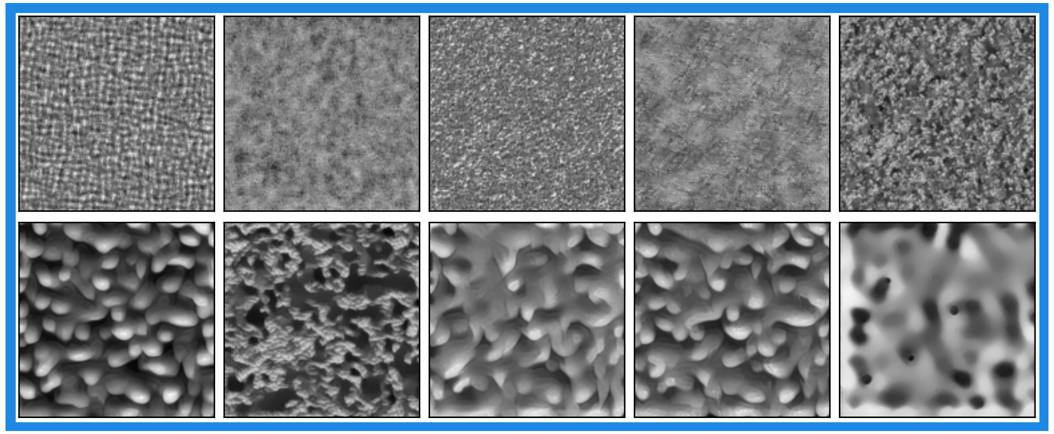
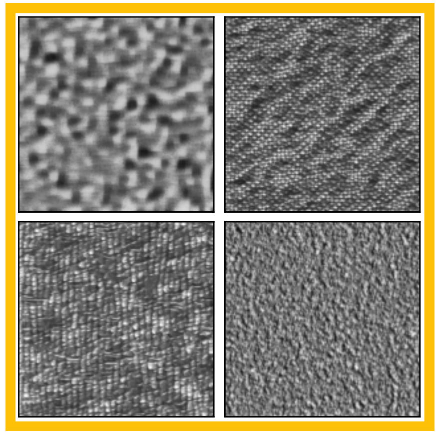
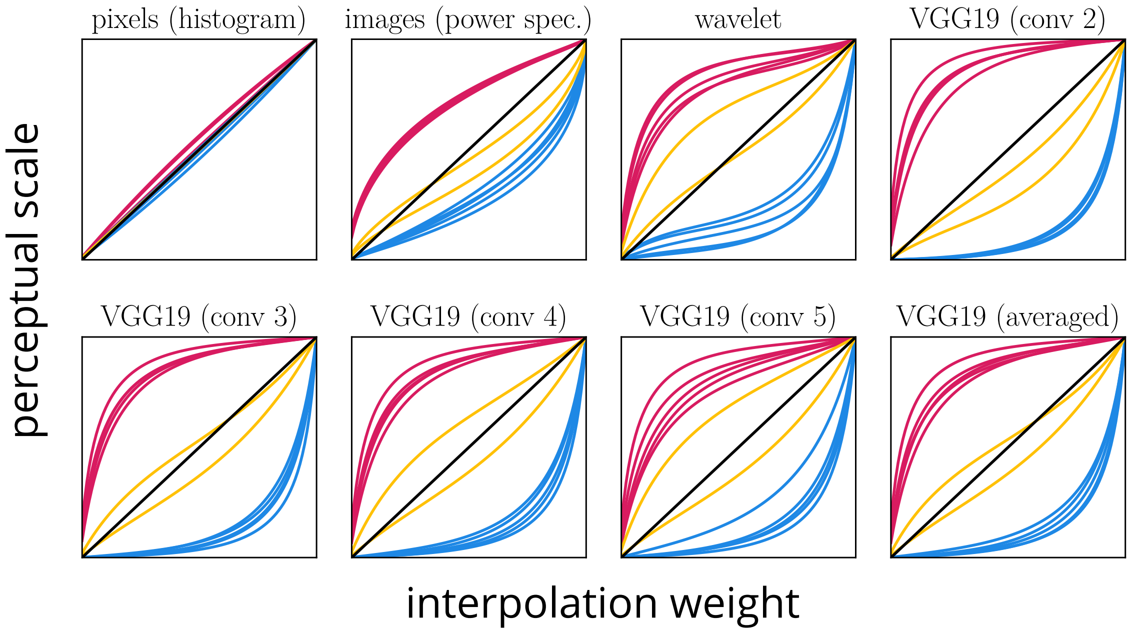
Predictions (Interp. of Naturalistic Textures)









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Results (Interp. of Naturalistic Textures)

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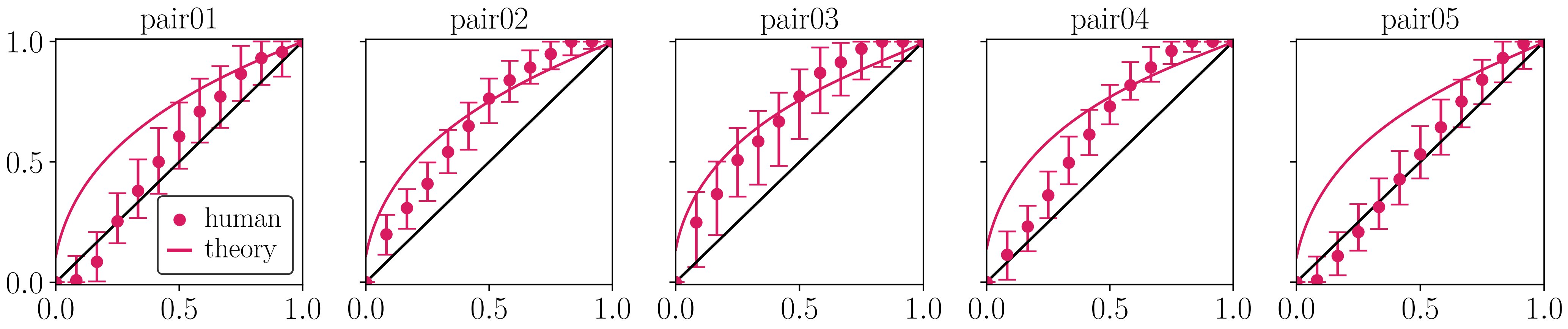
- Early sensitivity group



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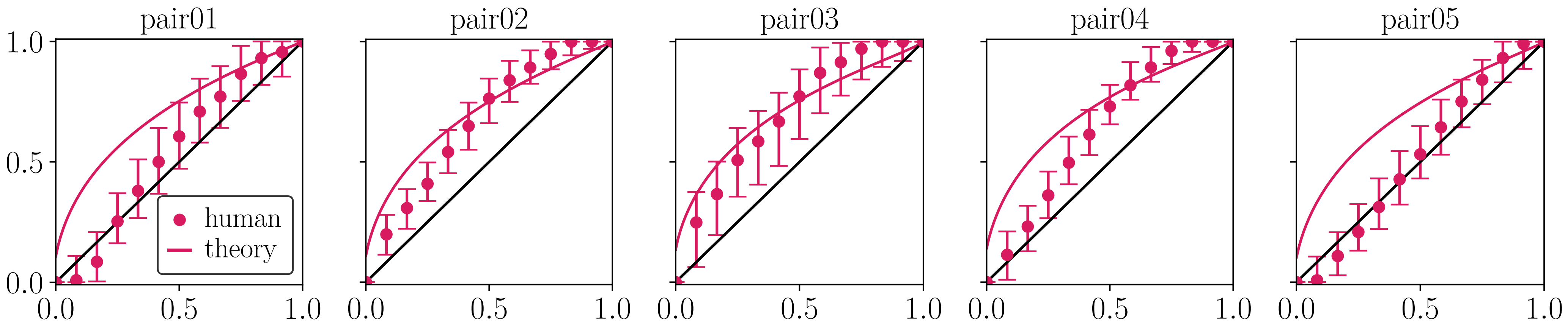
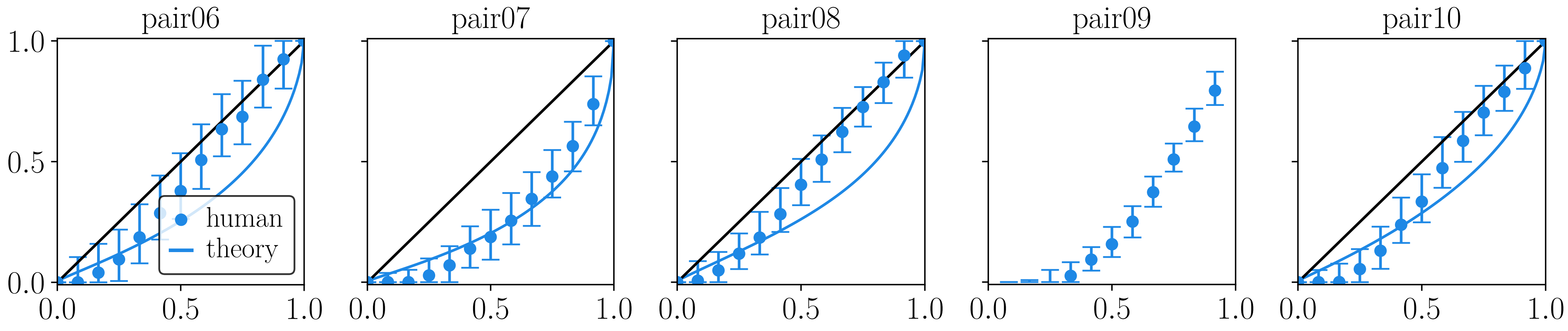


Late sensitivity group



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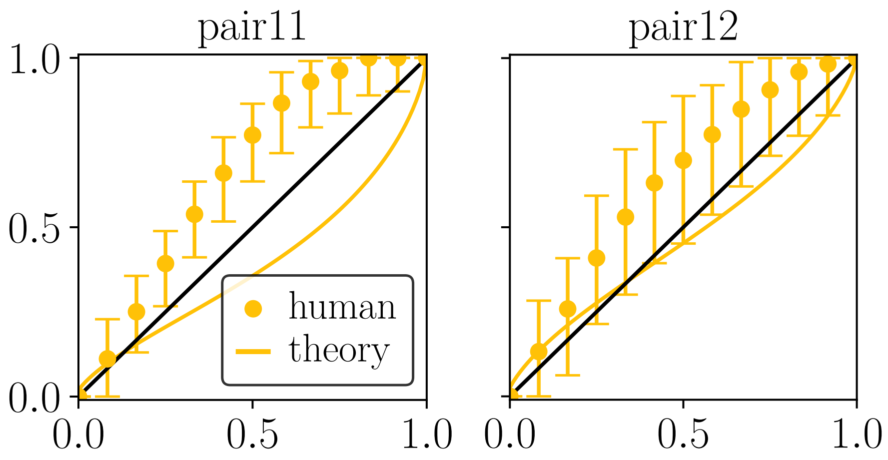
Results (Interp. of Naturalistic Textures)

- Conflicting prediction group



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Area Matching Score :

$$\mathrm{AMS} = \int_0^1 \frac{\mathrm{sign}(f_{\mathrm{m}}(x) - x)(f_{\mathrm{th}}(x) - x)}{|f_{\mathrm{m}}(x) - x|} \mathrm{d}x$$

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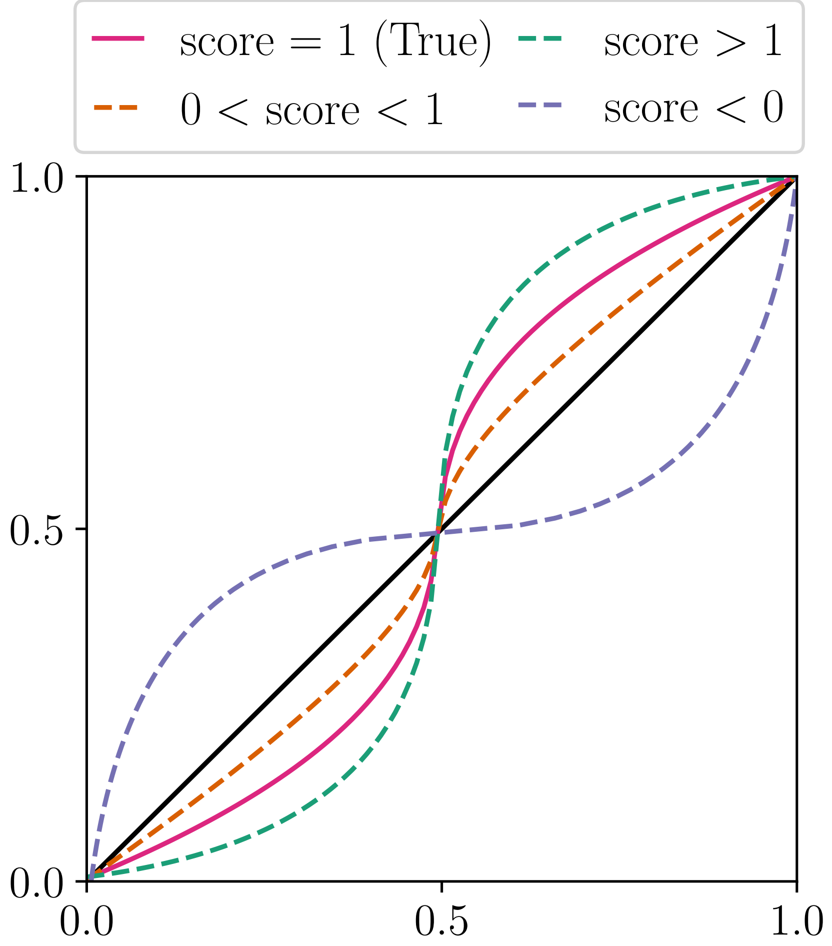

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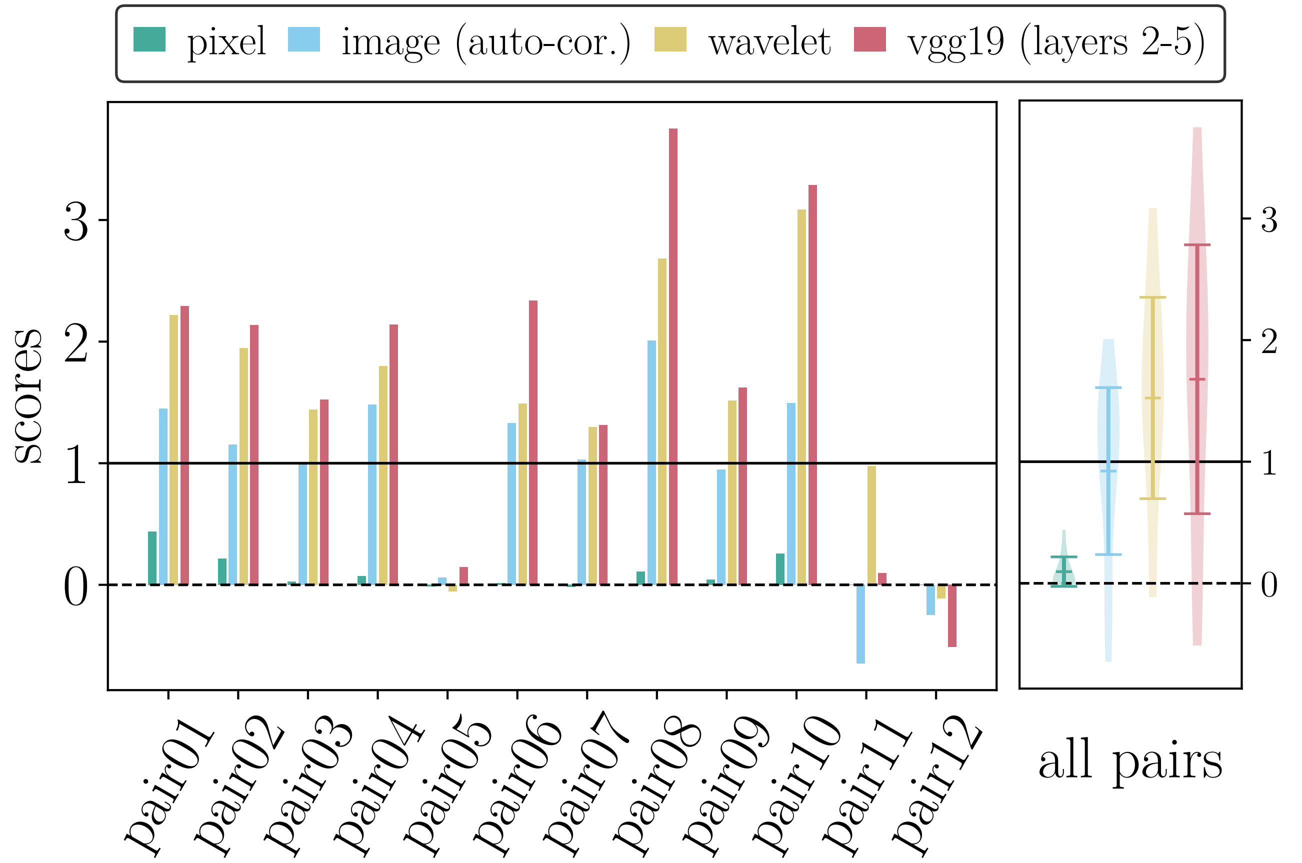

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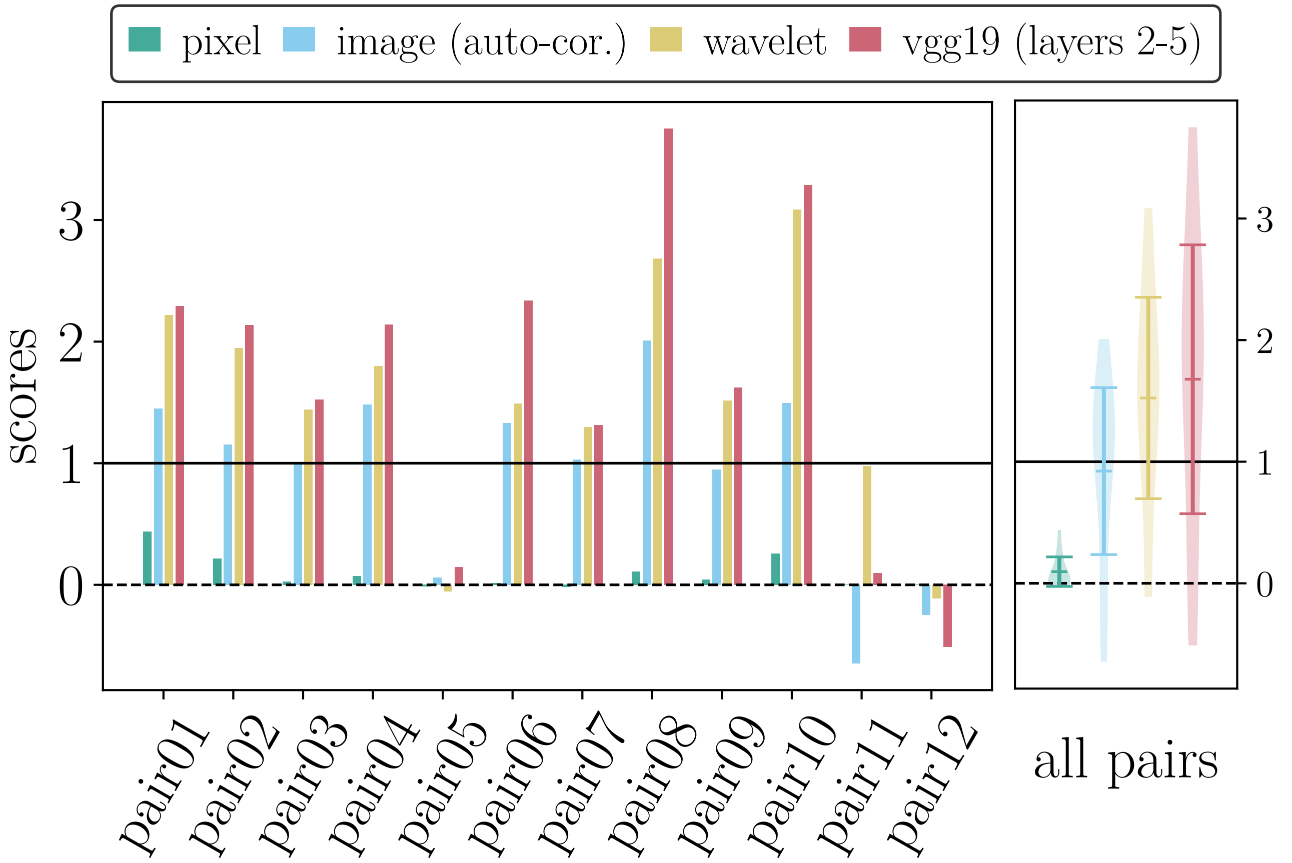

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- Gaussian textures : same prediction for different measurements hypothesis (Shot Noise local distributions *vs* Gaussian random field power spectrum)
- Interpolated naturalistic textures : require to make hypothesis about measurements $\Longrightarrow$ mostly driven by power spectrum
- Measure of perceptual path instead of distance : going towards measuring perceptual metric !!

Thanks for you attention !

Questions ?

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